Probability concepts

Exercise set 5

1. Which of these is a valid probability density function?

a) $f(x) = 1/3$ for $4.5 < x < 8.5$, 0 elsewhere

b) $f(x) = 2\sqrt{2}$ for $5\sqrt{2} < x < 5.5\sqrt{2}$, 0 elsewhere

c) $f(x) = -0.5$ for $8.2 < x < 10.2$, 0 elsewhere

d) $f(x) = x$ for $-1 < x < \sqrt{3}$, 0 elsewhere

e) $f(x) = x$ for $1 < x < \sqrt{3}$, 0 elsewhere

f) $f(x) = x$ for $0 < x < 2.5$, 0 elsewhere

2. X is a continuous random variable with the following density function

$$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ \\ 0 & \text{other values of x} \end{cases}$$

Find c and cumulative distribution function of X. Calculate probability that X is less then 0 and probability that X is between 0 and 1.

3. X denotes amount of time (in days) a value of an investment fund will stay above a benchmark. Probability density function of X is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ \\ 0 & \text{other values of x} \end{cases}$$

Find probability that the investment fund will stay above benchmark for at least 20 days.

4. In a portfolio there are seven investment funds as described in question 4. Assuming their performance is independent, what is the probability that three out of seven funds will stay above the benchmark for at least 20 days?

5. Find E(X) if density function of X is given by:

$$f(x) = \begin{cases} \frac{50}{x^3} & x > 5 \\ \\ 0 & \text{other values of x} \end{cases}$$

6. X is a continuous random variable with the following density function

$$f(x) = \begin{cases} ax^2 & 0 \leqslant x \leqslant 4 \\ 0 & \text{other values of x} \end{cases}$$

Find a, E(X) and variance of X.

7. X has a continuous uniform distribution in an interval $[-2, 16]$. Calculate $P(X < 0), P(X > 7), P(-1 < X < 1), E(X)$ and variance of X.

8. We are waiting for a bus at a bus stop at 8:00 am. The arrival time of the bus has uniform distribution between 8:00 and 8:20. Calculate the probability that we will wait less than 10 minutes and probability that we will wait more than 5 minutes. What is the expected number of minutes we will have to wait?

9. Trains going to Gdańsk arrive at Sopot train station every 15 minutes starting at 8:00 am, and trains going to Gdynia arrive every 15 minutes starting at 8:05 am. We arrive at the train station at a time uniformly distributed between 8:00 and 8:30 am and get on the first train that arrives. What is the probability that we will go to Gdańsk? How would the answer change if we arrived at a time unfirmly distributed between 8:10 and 8:40 am?