Continuous random variables

$P(a < X < b) = \int_a^b f(x)dx$

$P(X < a) = F(a) = \int_{-\infty}^a f(x)dx$

$E(X) = \int_{-\infty}^{\infty} xf(x)dx$

$E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx$

$Var(X) = E(X^2) - (E(X))^2$

Continuous uniform distribution

$f(x) = \frac{1}{b-a}$ for $a < x < b$, 0 otherwise

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leqslant x \leqslant b \\ 1 & x > b \end{cases}$$

$E(X) = \frac{a+b}{2}, Var(X) = \frac{(b-a)^2}{12}$

Normal distribution

$Z = \frac{X-\mu}{\sigma}$

- $P(Z > x) = 1 - P(Z < x)$
- $P(Z < -x) = P(Z > x)$
- $P(a < Z < b) = P(Z < b) - P(Z < a)$

Exponential distribution

$f(x) = \lambda e^{-\lambda x} \quad for \quad x \geqslant 0 \quad$ (0 otherwise)

$F(x) = 1 - e^{-\lambda x} \quad for \quad x \geqslant 0 \quad$ (0 otherwise)

$E(X) = \frac{1}{\lambda}, Var(X) = \frac{1}{\lambda^2}$

Memorylessness property: $P(X > t + s|X > t) = P(X > s)$

Joint distributions (discrete)

$p(x,y) = P(X=x, Y=y) \quad p_X(x) = \sum_y p(x,y) \quad p_Y(y) = \sum_x p(x,y)$

$E(XY) = \sum_y \sum_x xyp(x,y)$

Multinomial distribution

$P(X_1 = n_1, X_2 = n_2, ..., X_r = n_r) = \frac{n!}{n_1!n_2!...n_r!} p_1^{n_1} p_2^{n_2} ... p_r^{n_r}$

Joint distributions (continuous)

$P(a < X < b, c < Y < d) = \int_c^d \int_a^b f(x,y)dxdy \quad f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$ $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx$

$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xyf(x,y)dxdy$

Independent random variables

$p(x,y) = p_X(x)p_Y(y), \quad f(x,y) = f_X(x)f_Y(y)$

Covariance and correlation

Cov(X,Y) = E(XY)-E(X)E(Y)

$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$

Expected return on a portfolio

$E(R_p) = w_1E(R_1) + w_2E(R_2) + ... + w_nE(R_n)$

Portfolio variance

$\sigma^2(R_p) = w_1^2Var(R_1) + w_2^2Var(R_2) + w_3^2Var(R_3) + 2w_1w_2Cov(R_1,R_2) + 2w_1w_3Cov(R_1,R_3) + 2w_2w_3Cov(R_2,R_3)$

Integration

$\int_a^b x^n dx = [\frac{1}{n+1}x^{n+1}]_a^b = \frac{1}{n+1}b^{n+1} - \frac{1}{n+1}a^{n+1}$

$\int_a^b kdx = [kx]_a^b = kb - ka$