Principles of counting

Multiplication rule

$n_1 \times n_2 \times ... \times n_k$

Multinomial formula

$\frac{n!}{n_1! \times n_2! \times ... \times n_k!}$

Combinations

$nCr = \frac{n!}{r!(n-r)!}$

Permutations

$nPr = \frac{n!}{(n-r)!}$

Probabilities formulas

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$

$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$

$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)} = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^{n} P(A|B_j)P(B_j)}$

Discrete distributions

$E(X) = \sum_{x=1}^{n} x \; f(x)$

$E(g(X)) = \sum_{x=1}^{n} g(x) \; f(x)$

$Var(X) = E[(X - E(X))^2] = E(X^2) - (E(X))^2$

Binomial distribution

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$E(X) = np$

$Var(X) = np(1-p)$

Poisson distribution

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$E(X) = \lambda$

$Var(X) = \lambda$

Geometric distribution

$f(x) = (1-p)^{x-1} p$

$E(X) = \frac{1}{p}$

$Var(X) = \frac{1-p}{p^2}$

Negative binomial distribution

$P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$

$E(X) = \frac{r}{p}$

$Var(X) = \frac{r(1-p)}{p^2}$

Hypergeometric distribution

$P(X = x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$

$E(X) = \frac{nm}{N}$

$Var(X) = \frac{nm}{N}[\frac{(n-1)(m-1)}{N-1} + 1 - \frac{nm}{N}]$